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RESEARCH ARTICLE

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Instabilities of standing waves and positivity in traveling waves to a higher-order Schrödinger equation

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The aim of this paper is to explore a Schrödinger equation that incorporates a higher-order operator. Traditional models for electron dynamics have utilized a second-order diffusion Schrödinger equation, where oscillatory behavior is achieved through complex domain formulations. Incorporating a higher-order operator enables the induction of oscillatory spatial patterns in solutions. Our analysis initiates with a variational formulation within generalized spaces, facilitating the examination of solution boundedness. Subsequently, we delve into the oscillatory characteristics of solutions, drawing upon a series of lemmas originally applied to the Kuramoto–Sivashinsky equation, the Cahn–Hilliard equation, and other equations that employ higher-order operators. Specific solution types, such as standing waves, are numerically investigated to illustrate the oscillatory spatial patterns. The discussion then extends to the theory of traveling waves to establish general conditions for positive solutions. A contribution of this work is the precise evaluation of a critical traveling wave speed, denoted as c^*, above which the first minimum remains positive. For values of the traveling wave speed c significantly greater than c^*, the solutions can be entirely positive.

KEYWORDS
higher-order operator, instabilities, standing waves, traveling waves

MSC CLASSIFICATION
35Q41, 35B35

1 | PROBLEM DESCRIPTION AND OBJECTIVES

Diffusion is important in depicting various physical phenomena as described in [1]. The accurate portrayal of diffusion-driven phenomena necessitates the selection of an operator (in our case within the frame of Partial Differential Equations) or a combination thereof to accurately forecast particle movements. These portrayals can be rooted under several formulations highlighting statistical approaches (as discussed in [2] and the references therein) and mathematical formulations of motion energy. An illustrative instance is provided by [3] and [4], where the concept of free energy, initially introduced by Landau and Ginzburg, is utilized. The expression for the free energy presented in [3] is given by $\frac{1}{2}k(\nabla u)^2$ (with u representing concentration). Based on this free energy, the work [3] employs a chemical potential to derive a governing equation consisting on a fourth-order operator. A significant mathematical challenge associated with higher-order operators is the complexity of establishing a maximum principle, as highlighted in references [5–7].

Other ways of modeling diffusion consisted on the observation of a physical process and the proposal of an operator whose mathematical properties fit with the observations. There are plenty of examples in the literature, but we refer to the celebrated Keller and Segel model as an illustrative case. This model is used to understand chemotaxis processes in cells [8] (see as well the analyses in [9–12]). The proposal of dedicated operators for modeling purposes has been followed in

