Eur. Phys. J. Plus (2025) 140:516
https://doi.org/10.1140/epjp/s13360-025-06410-0

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

# Gradient descent and response surface optimisation for nonlinear dynamics over an unstable heated wedge: lie group and sensitivity analysis

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Received: 10 April 2024 / Accepted: 8 May 2025


**Abstract** Thermal conductivity and thermodynamic properties make nanofluids highly effective in thermal analysis and engineering applications. Ternary hybrid nanofluids flow over wedge surfaces can significantly enhance hydraulics and geothermal applications. This study explores a novel approach to optimising manufacturing processes in industries like plastic film production, heat exchangers, glass fibres, petroleum, polymer sheets, and electronic cooling systems. A key innovation of this work is the application of $H(OCH_2CH_2)_nOH–H_2O$ as a base fluid with AA7072 (nanofluid), $ZrO_2$ + AA7072 (hybrid nanofluid), and MgO + AA7072 + $ZrO_2$ (ternary hybrid nanofluid). The study investigates the heat transfer characteristics of these fluids as they flow over a wedge under the influence of various boundary conditions. Response surface methodology (RSM) and prediction through gradient descent-based machine learning is employed to optimise the thermal performance. The BVP4C solver in MATLAB is used to solve the governing equations numerically, and the gradient descent technique provides accurate predictions of the thermal behaviour through Python programming. From Table 3, the optimisation results indicate that the minimum value of 0.062983 is observed in case 2. In contrast, the maximum value of 13.5527 is recorded in case 3, demonstrating the significant impact of ternary hybrid nanofluids on heat transfer enhancement.

## 1 Introduction

According to recent scientific research, heat transfer utilisation is crucial in many industrial and manufacturing processes. Nanoparticles have been used as attractive components in new fluids to maximise heat transfer in automated production systems. The enhanced thermophysical characteristics of nanoparticles make them helpful in cooling heating exchange processes in various technical systems, including fusion reactions, metallic strips, welding equipment, chemical reaction cooling, etc. Produced with insulated effective nanoparticles, nanofluids are liquids that transmit heat efficiently. Their versatility stems from the fact that they have unveiled novel thermophysical properties and capacities. When combined with a base liquid, nanofluids composed of particles on the nanometer scale improve thermal performance. The base fluid can be any of the following: ethanol, water, oil, or lubricant, among other options. Nanofluid selection is affected by nanoparticle size, shape, concentration, stability, and thermophysical characteristics. Research into nanofluid flow issues has attracted much attention from academics and industry professionals because of the wide variety of technical and commercial processes that may benefit from this material. Khan et al. [1] compared exponential and linear stretching sheets with two stratifications in a spinning Maxwell nanofluid flow. Kumar et al. [2] examined a theoretical and computational study of Casson nanofluid flow driven by chemical reactions and a magnetic region across the surface of stretched curved. Anwar et al. [3] examined nanoflow through a computer simulation of generated non-Newtonian MHD flow on a nonlinear stretched plane. Ramzan et al. conducted a partially ionised entropy minimisation study on Casson nanoflow across a bidirectionally stretched sheet

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Published online: 11 June 2025